

## A Manipulator Design Optimization Based on Constrained Multi-objective Evolutionary Algorithms

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**Abstract**—In order to achieve high performance of robotic manipulators, light-weight and high manipulability are two essential indicators. In this paper, a model of UR5 (Universal Robot) manipulator is defined and analyzed with focus on optimizing the total mass and manipulability. The kinematics and dynamics for the robotic manipulator are used to calculate the joints torque and define quantitative measures of manipulability. Then, a design optimization problem is formulated for UR5 manipulator. It adopts the weight and manipulability of the manipulator as the objective functions. The drive train constraints associate with joint motors and gearboxes have also been considered. Finally, constrained multi-objective evolutionary algorithms (CMOEAs) are employed to solve the formulated problem. Several reasonable optimal combinations of geometrical parameters and type selection of motor and gearbox are provided. And compare them with the original structure of UR5.

**Keywords**-Design automation, industrial robot, light-weight, manipulability, Constrained Multi-objective Evolutionary Algorithms

### I. INTRODUCTION

From the International Federation of Robotics statistics [1], it can be seen that the demand for industrial robots has accelerated considerably due to the ongoing trend toward automation and the continued innovative technical improvements of industrial robots. The researchers of industrial robots have made fruitful achievements and successful application cases. However, some design issues of robotic manipulators still challenge researchers. More effective methods for manipulator arms design and optimization are urgently explored. Especially, during the process of designing a manipulator, how to select suitable motors and gearboxes, whether the designed shape and size of a link are optimized and how to get high manipulability are a series of issues to be considered. Many researches have been done on those problems which guarantees the optimal performances of the integrated robotic system.

Many researchers [2][3][4] optimized reducers selection in system design process according to design specifications. Pettersson and Olvander [2] presented an optimization strategy to design industrial manipulators transmission chain. The gearboxes were simplified into an equivalent model

with mass, moment of inertia and friction. Zhou et al. [5] described a new approach to the design of a lightweight robotic arm for service applications. A major design objective is to achieve a lightweight robot with desired kinematic performance and compliance. This is accomplished by an integrated design optimization approach, where robot kinematics, dynamics, drive-train design and strength analysis by means of finite element analysis (FEA) are generally considered. Wang et al. [6] studied the optimum shape design of flexible manipulators under a specified total weight constraint. The optimization design problem was constructed using the geometrical dimension as the design variables and the fundamental frequency as the objective respectively. Besides, optimal and automated design of industrial robots has been addressed by Linkoping University and ABB [7][8][9]. Tarkian et al. [10] presented a multidisciplinary design optimization (MDO) framework for automated design of a modular industrial robot. The developed design framework seamlessly integrated high level computer aided design (CAD) templates (HLCT) and physics based high fidelity models for automated geometry manipulation, dynamic simulation, and structural strength analysis.

Estimating and optimizing the performance of manipulators is important in both theoretical studies and practical applications. The concept of manipulability of Yoshikawa [11] is based on the ability to posit and re-orientate the end-effector of the robotic arm in different directions. Optimizing the manipulability leads to increased performance for a robotic structure. The improvements in manipulability results in more versatile and supple robotic arms, which is pursued by the industry, also mentioned in [12].

This study focuses on optimizing the total mass and manipulability of a robotic arm based on CMOEAs. The rest of this paper is organized as follows. In Section II, kinematics and dynamics model of UR5 are defined. The design optimization is formulated in Section III. A design optimization experiment is implemented and several reasonable optimal solutions are shown out in section IV. In Section V, the paper is concluded.

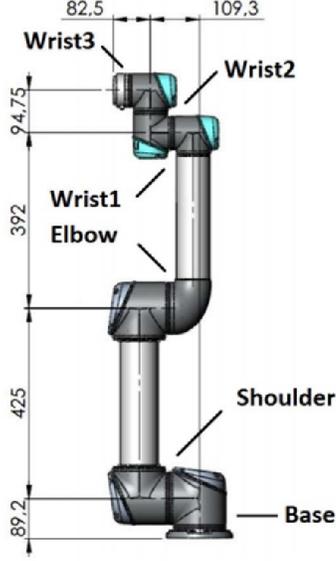


Figure 1. The Structure of UR5

## II. MODELING OF THE UR5 MANIPULATOR

Fig. 1 is extracted from UR5s user manual, which shows a sketch of the UR5 manipulator with its joints and links. The measurements of the size of the links are given by the manufacturer. The manipulator has six revolute joints  $j_i (i = 1, 2, \dots, 6)$  and six links  $l_i (i = 1, 2, \dots, 6)$ . Each revolute joint has one DOF, so UR5 has a total of six DOF. In order to derive the forward kinematics, the DenavitHartenberg (DH) parameters should be set in advance.

### A. Kinematics

The forward kinematics of the robotic arm is formulated based on the DH convention [13]. The coordinate frames  $o_i x_i y_i z_i (i = 1, 2, \dots, 6)$  are assigned based on the sketch of UR5. In Fig. 2, the coordinated frames are assigned, and the DH parameters are defined as listed in Tab. I.

With the DH parameters, the forward kinematics can be calculated by equation (1).

$$T_6^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5 \quad (1)$$

Where  $T_i^{i-1}$  denotes the homogeneous transformation from frame  $i-1$  to frame  $i$ , and it defined as follows.

$$A = \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, i = 1, 2, \dots, 6 \quad (2)$$

Where  $c$  and  $s$  are the abbreviation of  $\cos$  and  $\sin$  respectively.

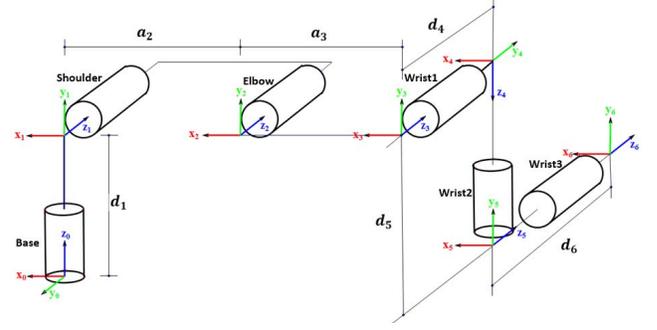


Figure 2. D-H Convention Frames Assignment of UR5

Table I  
D-H PARAMETERS

Link	$a_i (m)$	$\alpha_i (rad)$	$d_i (m)$	$\theta_i (rad)$
1	0	$\pi/2$	0.0892	$q_1$
2	-0.425	0	0	$q_2$
3	-0.395	0	0	$q_3$
4	0	$\pi/2$	0.1093	$q_4$
5	0	$-\pi/2$	0.0948	$q_5$
6	0	0	0.0825	$q_6$

### B. Dynamics

Dynamics is the study of how forces and torques impact the motion of an object. In mathematical terms, this relationship is presented using the motion equations. The computation of the inverse dynamics is a prerequisite for evaluating any given designs with given load and prescribed trajectory. In this section, we use the Euler-Lagrange method to represent the dynamical models of UR5. It is based on the difference between the kinetic energy  $K$  and the potential energy  $P$  of the manipulator.

$$L = K - P \quad (3)$$

Equation (3) is called the Lagrangian. Once the Lagrangian is calculated, the Euler-Lagrange equation can be used to find the equations of motion, which is shown as follows.

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = \tau_j, j = 1, 2, \dots, 6 \quad (4)$$

For UR5, the kinetic energy  $K$  and the potential energy  $P$  are given by the following two equations.

$$K = \dot{q}^T \sum_{i=1}^6 [m_i J_{v_i}^T J_{v_i} + J_{w_i}^T R_i I_i R_i^T J_{w_i}] \dot{q} \quad (5)$$

$$P = \sum_{i=1}^6 m_i g^T r_{c_i} \quad (6)$$

Where  $m_i$  is the mass of link  $i$ ,  $J_{v_i}$  and  $J_{w_i}$  are the Jacobian matrices to the center of mass of link  $i$ ,  $R_i$  is the rotation matrix from the inertial frame to frame  $i$ ,  $I_i$  is the inertia tensor of frame  $i$ ,  $g$  is the gravity vector in the inertial frame, and  $r_{ci}$  is the coordinate of the center of mass of link  $i$ . These dynamics equations are used in the calculation of joints torque, and the details will be presented in the following section.

### III. FORMULATION OF DESIGN PROBLEM

In this work, two optimization objectives have been considered. One is the total mass of the manipulator and the other one is the manipulability. A light weight robot can be obtained by minimizing the total mass. Maximizing the manipulator so that the manipulator achieve better performance. Meanwhile, these two objectives should meet all constraints associated with the motors and gearboxes simultaneously.

#### A. Manipulability

There exist certain points where the kinematic Jacobian matrix is not full rank in the joint space. These points are called singular points and the movement of the joint is limited to fewer dimensions. In [15], it is discussed that in the vicinity of singular points small velocities in the workspace leads to very large velocities in the joint space, and this is not preferable. Thus it is very important to avoid singular points, where the manipulability is greatly reduced. The purpose of manipulability indices is to give a quantitative measure of the ability to move and apply forces in arbitrary directions. It should also give information about the proximity of singular configurations. The manipulability measurement is based on the Jacobian matrix,  $J$  is defined [11] as follow.

$$\mu(q) = \sqrt{\det(J(q)J(q)^T)} \quad (7)$$

For the specific configuration of joints, larger values of  $\mu(q)$  leads to greater freedom, where  $q$  represent joints angles.

A commonly used measure of manipulability is the so called a velocity manipulability ellipsoid. The expression for the velocity manipulability ellipsoid is derived below. A unit sphere which considers the set of joint velocities with constant unit norm in the joint velocity space is stated as follow.

$$\dot{q}^T \dot{q} = 1 \quad (8)$$

For an  $N \times links$  robot, the Jacobian is a  $6 \times N$  matrix and a square Jacobian requires a robot with 6 joints. We can invert  $v = J(q)\dot{q}$  and write as follow.

$$\dot{q} = J(q)^{-1}v \quad (9)$$

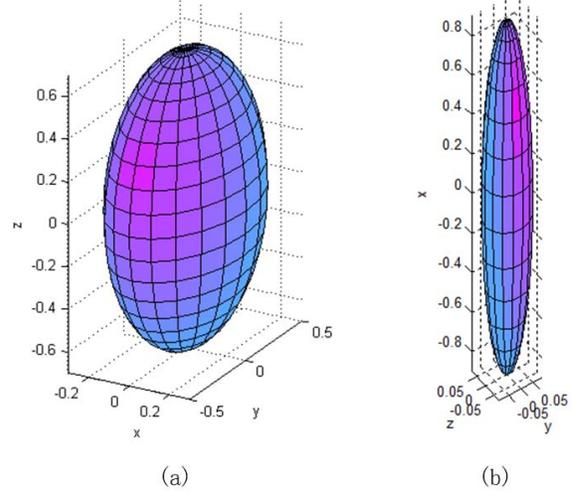


Figure 3. End-effector velocity ellipsoids: (a) Translational velocity ellipsoid for nominal pose, (b) rotational velocity ellipsoid for a near singular pose.

Where  $J(q)$  is square and non-singular and  $\dot{q}$  is the differential of joint angles  $q$ .  $v$  represents a spatial velocity of end-effector and it comprises translational and rotational velocity components. Combining (8) and (9) we can write as follow.

$$v^T (J(q)J(q)^T)^{-1}v = 1 \quad (10)$$

Which is the equation of points on the surface of a 6-dimensional ellipsoid in the end-effector velocity space, as shown in Fig. 3. If this ellipsoid is close to spherical, the end-effector can achieve arbitrary Cartesian velocity. Otherwise, the end-effector cannot achieve velocity in the directions corresponding to those small radius.

#### B. Drive Train Constraints

The drive train consist of motors, gears connecting with links. To select drive train as modular joint of a robot, the required torque of motor for each joint can be represented [2] by the following equation.

$$\tau_{m,i} = \left\{ (J_m + J_g)\ddot{q}(t)\rho + \frac{\tau(t)}{\eta\rho} \right\}_i, i = 1, 2, \dots, 6 \quad (11)$$

Where  $J_m$  is the inertia of motor shaft and  $J_g$  represents the equivalent inertia of gearbox reflected at motor shaft,  $\rho$  represents the gear ratio,  $\ddot{q}(t)$  are the angular acceleration of links,  $\tau(t)$  are the required torque of links, which can be calculated by (4), and  $\eta$  is the efficiency of drive train. Based on the required torque of motor, the equivalent required torque is given as follow.

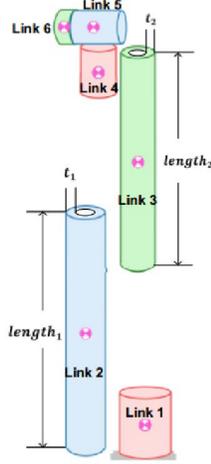


Figure 4. Sketch of UR5

$$\tau_{rmsi} = \left\{ \sqrt{\frac{1}{\delta t} \int_0^{\delta t} \tau_m^2 dt} \right\}_i, i = 1, 2, \dots, 6 \quad (12)$$

For the selection of motors and gearbox, there are often some criteria [4] represented by the following inequations.

$$T_m \geq \tau_{rms}, T_g^{max} \geq \tau_p, N_m^{max} \geq n_p, N_g^{max} \geq n_{gp} \quad (13)$$

where the  $\tau_{rms} = \frac{1}{\delta t} \int_0^{\delta t} \tau_m^2 dt$ , with  $\delta t$  being the duration of a characteristic working cycle.  $\tau_p = \max\{|\tau(t)|\}$  is the peak torque of motor.  $n_p$  and  $n_{np}$  are the speed of motors and gearboxes according to the requirement of the robotic arm and gear ratio respectively.  $T_m$  is the nominal torque of motor, and  $N_m^{max}$  and  $N_g^{max}$  are the maximum speed of the motor and gearbox respectively.

### C. Objective Functions Formulation

A sketch of UR5 is shown in Fig. 4. Two main links, link 2 and link 3, are hollow cylinder.  $length_1$  and  $length_2$  are the length of link 2 and link 3 respectively.  $t_1$  and  $t_2$  represent the thickness of Link2 and Link 3 respectively. They are also design variables. In order to conveniently calculate the total mass of the manipulator, each links mass center is approximated to its geometric center. The symbol  $m_{body}$  represents the total mass of UR5 without motors and gearboxes.

The first objective is to minimize the total mass of the manipulator. The second objective is to maximize the manipulability of the manipulator. Therefore, the optimization task is to find the lightest combination of motor and gearbox for all six DOF and the optimal length and thickness of Link 2 and Link 3 that fulfill all constraints associated with

the motors and gearboxes. The objective function  $f_1(x)$  is defined as the sum of the mass of body and the drive train, as shown in (14). The objective function  $f_2(x)$  is defined as the manipulability of UR5 in a given trajectory, as shown in (15). In addition, we invert the maximization optimization problem to minimize the opposite number of total manipulability. Equations (17-20) represent the whole constraints associate with motors and gearboxes. In general, the above two objectives are conflicting.

$$\min_x f_1(x) = m_{body} + \sum_{i=1}^6 m_m(u_m) + m_g(u_g) \quad (14)$$

$$\min_x f_2(x) = - \sum_{i=1}^N \det(J(x)J(x)^T) \quad (15)$$

$$x = [length_1, length_2, t_1, t_2, u_m, u_g] \quad (16)$$

The constraints are present as follows.

$$T_{m,i} \geq \sqrt{\frac{1}{\delta t} \int_0^{\delta t} \left\{ (J_m(x) + J_g(x)) \ddot{q}(t) \rho + \frac{\tau(t, x)}{\rho \eta_g} \right\}_i^2} \quad (17)$$

$$T_{m,i}^{max} \geq \max\{|\tau(t, x)|\}_i \quad (18)$$

$$N_{m,i}^{max} \geq \max\left\{ \left| \frac{60}{20\pi} \dot{q}(t) \rho \right| \right\}_i \quad (19)$$

$$N_{g,i}^{max} \geq \max\left\{ \left| \frac{60}{20\pi} \dot{q}(t) \rho \right| \right\}_i \quad (20)$$

Where the design variables  $x$  includes both the manipulators base size design variables  $length_1, length_2, t_1$  and  $t_2$  and the motors  $u_m = [u_{m1}, \dots, u_{m9}]$  and gearboxes  $u_g = [u_{g1}, \dots, u_{g4}]$ . So far, we have formulated the design problem as a discrete and continuous mixed constrained multi-objective optimization problem, which can be solved by CMOEAs.

## IV. DESIGN OPTIMIZATION EXPERIMENT

A design optimization is conducted on UR5 robotic arm. The structure of the manipulator is fixed. Except Link 2 and Link3, every links length is fixed. The design optimization problem is formulated in Section III, including the objective functions, design variables and drive train constraints.

Table II  
CANDIDATE MOTOR DATA

Motor Type	$T_m$ (Nm)	$T_m^{max}$ (Nm)	$N_m^{max}$ (rpm)	$J_m$ (gcm <sup>2</sup> )	$m_m$ (kg)
EC-i40(70W)	0.0667	1.81	15000	24.2	0.21
EC-32(80W)	0.0426	0.353	25000	20	0.27
RE-35(90W)	0.0965	0.967	12000	67.4	0.34
EC-40(100W)	0.127	0.94	18000	85	0.39
RE-40(150W)	0.184	2.5	12000	138	0.48
EC-40(170W)	0.165	2.66	18000	53.8	0.58
EC-i52(180W)	0.366	15	6000	141	0.82
EC-45(150W)	0.186	0.872	15000	119	0.85
EC-32(200W)	0.0405	8.9	9500	560	1.1

Table III  
CANDIDATE GEARBOX DATA

Gear Type	$T_g$ (Nm)	$T_g^{max}$ (Nm)	$N_g^{max}$ (rpm)	$J_g$ (kgm <sup>2</sup> )	$m_g$ (kg)
HPGP-14	30	56	6000	$2 \times 10^{-6}$	0.63
HPN-14	50	110	6000	$9 \times 10^{-6}$	0.95
HPG-20A	100	217	6000	$21 \times 10^{-6}$	1.6
HPGP-20	133	217	18000	$17 \times 10^{-6}$	1.9

### A. Experiment Describe

The two objective functions focus on the total mass and manipulability of UR5. Design variables  $x$  include both the manipulators base size variables and the indexes of motors and gearboxes.

The total mass of manipulator depends on the size and material of every link and the types of motors and gearboxes.  $m_{body}$  represents the mass of the body except drive train. The total mass also includes the part, the weight of joint motors and gearboxes. Tab. II gives the candidate Maxon motor data. Tab. III gives the candidate harmonic gearbox data.

In order to conveniently calculate the manipulability objective function. A trajectory of the end-effector of manipulator should be defined. In this experiment, we define an end-effector trajectory in Cartesian space. Then we invert it to joint trajectory in joint space. The motion of every joint is shown in Fig. 5. The runtime is set to  $T = 3s$ . Then we averagely divide it into  $N$  periods. Here, we set  $N = 54$ .

### B. Experiment Results and Analysis

In this work, CMOEAs are employed to solve the formulated problem. The CMOEAs have been used is non-dominated sorting genetic algorithm II (NSGA-II) [16] and adaptive tradeoff model for constrained evolutionary optimization (ATM) [17]. Population size for these approaches have been set to 200 and the generations have been set to

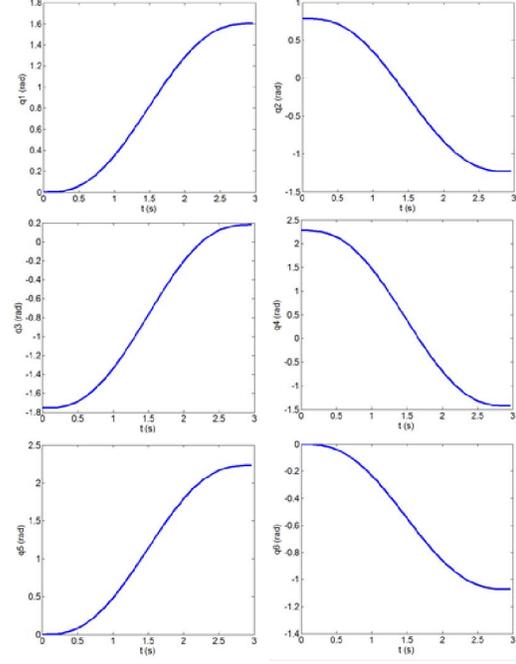


Figure 5. Trajectory in joint space

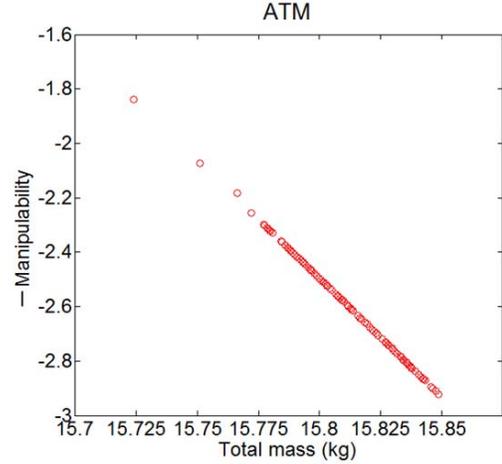


Figure 6. Pareto front of ATM

500. The 1st order Pareto fronts of ATM, NSGA-II and SP are visualized in Fig. 6 and Fig. 7 respectively. As can be seen, both the strategies have found optimal solutions. Via analyzing the optimization process shown in Fig. 6 and Fig. 7, it can be seen that the relationship between the optimization objectives 1 and 2 are shown nearly piecewise linear.

The second objective is the manipulability of UR5 for a given trajectory. We invert the maximization optimization problem to minimize the opposite number of total manipulability. So the negative value of the manipulability is shown

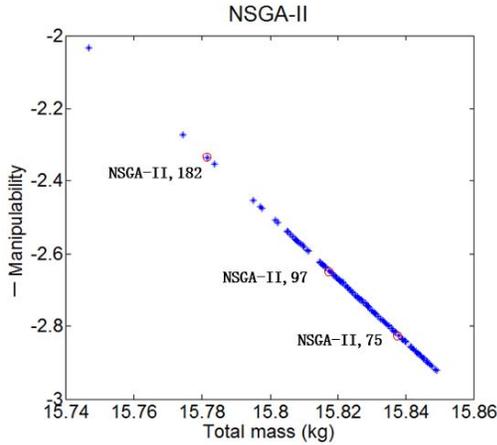


Figure 7. Pareto front of NSGA-II

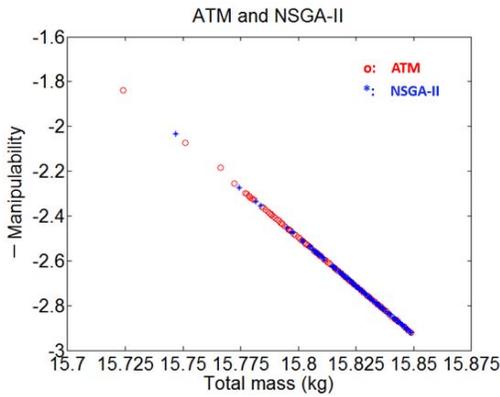


Figure 8. Pareto fronts of the ATM and NSGA-II

in Fig. 6 and Fig. 7. The convergence of the objective function is depicted in Fig. 8. After 500 iterations, the Pareto front of these two approach are very similar, which is shown in Fig. 8. The optimized weight of the robotic arm ranges from 15.7kg to 15.9kg. While the optimized manipulability of the robotic arm ranges 1.5 to 3. Both these two objectives have a small value range. This is due to the narrow candidate set of motors and gearboxes. Although all these algorithms converge to the optimal solutions, some of these solutions are not in accordance with the reasonable parameters in the process of manipulator. In order to obtain better solutions, more constraints should be considered. Especially the stiffness of every link, the load of end-effector and the life time of manipulator are very important factors of design optimization of robotic arm.

The two objectives is mutually non-dominated. So those solutions could not be directly compared. The selected final solution could depend on experiences and actual design requirements. Such as, the motor in base generally have larger torque than motor in wrist, the thickness of link should

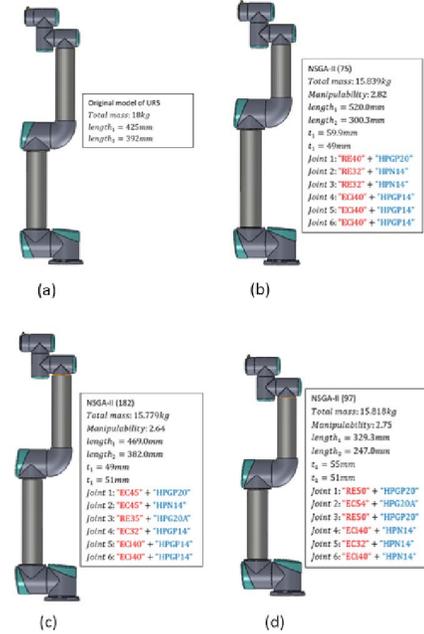


Figure 9. Optimal robot variants from NSGA-II Pareto front

not be too thin and so on. Several optimal combinations of geometrical parameters and type selection of motor and gearbox are provided, which is marked in Fig. 7. The 182nd, 97th and 75th population individual of NSGA-II are selected as examples compare with the original structure of UR5. Which is shown in Fig. 9.

## V. CONCLUSION

In the process of design manipulator, weight and manipulability are two important indicators should be considered. The manipulator geometric parameters design and motor and gearbox selection are optimized simultaneously, which formulates a discrete and continuous mixed optimization problem. Constraints are formulated by considering both motor and gearbox characteristics and robotic arm dynamics. A design optimization is conducted on UR5 robotic arm, which is solved by CMOEAs. It is able to reach a design with lower mass and higher manipulability for a given set of driving components. Which obtains several optimal combinations of geometrical parameters and types of motor and gearbox.

However, there are some important points arent mentioned in the optimization. Such as finite element analysis (FEA) and stiffness analysis and so on. Our future work may include the implement of a co-simulation platform consisting of ADAMS dynamics model and optimization algorithms. Which will enables design optimization based on dynamics of an embodiment existing in CAD systems. Besides, more objectives and also the arm morphology will be considered in the future.

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#### REFERENCES

- [1] International Federation of Robotics. Industrial robots: Statistics. <http://www.ifr.org>. Accessed May 18, 2015.
- [2] Pettersson, M., & Olvander, J. (2009). Drive train optimization for industrial robots. *IEEE Transactions on Robotics*, 25(6), 1419-1424.
- [3] Giberti, H., Cinquemani, S., & Legnani, G. (2010). Effects of transmission mechanical characteristics on the choice of a motor-reducer. *Mechatronics*, 20(5), 604-610.
- [4] Zhou, L., Bai, S., & Hansen, M. R. (2011). Design optimization on the drive train of a light-weight robotic arm. *Mechatronics*, 21(3), 560-569.
- [5] Zhou, L., & Bai, S. (2015). A new approach to design of a lightweight anthropomorphic arm for service applications. *Journal of Mechanisms and Robotics*, 7(3), 031001.
- [6] Wang F. Y. and Russell J. L., Optimum shape construction of flexible manipulators with total weight constraint, [J]. *IEEE Transactions on Systems, Man and Cybernetics*, 25(4): 605-614, 1995.
- [7] Bjorn, J., Olvander, J., & Pettersson, M. (2007). Component Based Modelling And Optimization For Modular Robot Design. In *ASME 2007 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 911-920). ASME.
- [8] Pettersson, M., Andersson, J., & Krus, P. (2005, January). Methods for discrete design optimization. In *ASME 2005 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 295-303). American Society of Mechanical Engineers.
- [9] Tarkian, M., Olvander, J., Feng, X., & Petterson, M. (2009, January). Design automation of modular industrial robots. In *ASME 2009 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference* (pp. 655-664). American Society of Mechanical Engineers.
- [10] Tarkian, M., Persson, J., Olvander, J., & Feng, X. (2012). Multidisciplinary design optimization of modular industrial robots by utilizing high level CAD templates. *Journal of Mechanical Design*, 134(12), 124502.
- [11] T. Yoshikawa, Manipulability of robotic mechanisms, *International Journal of Robotics Research*, 4(2): 3-9, 1985.
- [12] Kamrani, B., Wappling, D., Andersson, H., Berbyuk, V., & Feng, X. (2010). Optimal usage of robot manipulators (pp. 1-26). INTECH Open Access Publisher.
- [13] Denavit, J. (1955). A kinematic notation for lower-pair mechanisms based on matrices. *Trans. of the ASME. Journal of Applied Mechanics*, 22, 215-221.
- [14] P. I. Corke, *Robotics, Vision & Control*, Springer, ISBN 978-3-642- 20143-1, 2011.
- [15] B. Siciliano, L. Sciavicco, L. Villani and G. Oriolo, *Robotics: Modelling, Planning and Control*, Springer, second edition, 2011.
- [16] Deb K., Pratap A., Agarwal S., Meyarivan T., A fast and elitist multiobjective genetic algorithm: NSGA-II, *Evolutionary Computation*, *IEEE Transactions on*, vol.6, no.2, pp.182-197, Apr 2002.
- [17] Wang, Y., Cai, Z., Zhou, Y., & Zeng, W. (2008). An adaptive tradeoff model for constrained evolutionary optimization. *IEEE Transactions on Evolutionary Computation*, 12(1), 80-92.